

$D_s(0^\pm)$ Mesons spectroscopy in Gaussian Sum Rules *

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The masses of the $D_s(0^\pm)$ mesons are investigated from a view-point of ordinary light-heavy system in the framework of the Gaussian sum rules, which are worked out by means of the Laplacian transformation to the usual Borel sum rules. Using the standard input of QCD non-perturbative parameters, the corresponding mass spectra and couplings of the currents to the $D_s(0^\pm)$ mesons are obtained. Our results are $m_{D_s(0^-)} = 1.968 \pm 0.016 \pm 0.003$ GeV and $m_{D_s(0^+)} = 2.320 \pm 0.014 \pm 0.003$ GeV, which are in accordance well with the experimental data, 1.969 GeV and 2.317 GeV.

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Recently, the experimental observations of D meson and the corresponding theoretical manipulations attract much attention in the research of particle physics[1–10]. Many Quantum Chromodynamics (QCD) calculations about D mesons are in good agreement with the experimental data within the theoretical uncertainty except for the unexpected low mass of $D_{0^+}(2317)$ [11–14]. In Ref. [15], the masses of the excited $(0^+, 1^+)$ and $(1^+, 2^+)$ doublets for the $c\bar{s}$ system are calculated to the $1/m_c$ order in the sum rules based on heavy quark effective theory(HQET), and the $D_s(2317)$ and $D_s(2460)$ are identified as the $(0^+, 1^+)$ doublet so that the mass splitting in this doublet is well reproduced. We note that most of these theoretical calculations are based on QCD Borel sum rule (BSR) which emphasize the contributions of lowest resonance state, and have shown the power in the investigation of the non-perturbative properties of hadron bound states. However, excited states and continuous spectrum can produce background interference. Generally, this background interference increases with the mass of the hadron state considered, and this restrict the application range of BSRs. It is noticed that the QCD Gaussian sum rule (GSR) developed later emphasizes only the contribution of the hadron state considered as seen from the appearance of the Gaussian distribution function, and has more clean background in comparison with the BRS. From this reason, the GSR may, in principle, work better than the BSR. At least, both sum rules should give almost the same results because they are derived from the same underlying dynamical theory.

On the other hand, whether the D_s meson's spectroscopy observed by experiments can be derived out from the elementary theory of the strong interactions is a key point for testing the QCD non-perturbative mechanism, and serves also to be the starting point of calculating the important physical processes, such as the D_s meson decays and etc, from the first principle.

However, the results obtained by using the BSRs, $m_{D_s(0^+)} = 2.48 \pm 0.03$ GeV and $m_{D_s(0^-)} = 1.94 \pm 0.03$ GeV [11], are obvious inconsistent with the experiment data, $m_{D_s(0^+)} = 2.317 \pm 0.0013$ GeV[8] and $m_{D_s(0^-)} = 1.969$ GeV. For checking the correctness of QCD and exploring the non-perturbative mechanism, it's necessary to do a further study.

In order to make a cross check between various types of QCD sum rules, namely the BSRs[11] and the HQET sum rules[15], we calculate the masses m_{D_s} of D_s mesons and the couplings f_{D_s} of the corresponding currents to these mesons by using GSRs.

Consider the gauge-invariant and Lorentz-covariant two-point function of the current $J_i(x)$ corresponding to a resonance with the quantum numbers $i = 0^-, 0^+$ [16, 17]

$$\Pi_i(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T J_i(x) J_i^\dagger(0) | 0 \rangle, \quad (1)$$

where $|0\rangle$ is nonperturbative QCD vacuum, and $J_i(x)$ are the pseudoscalar and scalar currents

$$J_{0^-}(x) = i\bar{q}(x)\gamma_5 c(x), \quad (2)$$

$$J_{0^+}(x) = \bar{q}(x)c(x), \quad (3)$$

with $q(x)$ and $c(x)$ being the strange-quark and charm-quark fields at the point x , respectively. For the invariant functions $\Pi(q^2) = \Pi_i(q^2)$ ($i = P, S$), we have a dispersion relations without any subtraction

$$\Pi(q^2) = \frac{1}{\pi} \int ds \frac{\text{Im}\Pi(s)}{s - q^2 + i\epsilon}. \quad (4)$$

Via this dispersion relation, the QCD sum rule (QSR) was constructed by equating the contribution of operator product expansion (OPE) and the phenomenological (PH) one related to the spectral function. The former is described as the product of Wilson coefficients and non-perturbative QCD vacuum condensates or quark masses, while the latter is parameterized by hadronic quantities such as resonance masses, couplings and the continuum threshold, etc. Thus, the QSR can be represented in a simple form,

$$\int_{m_c^2}^{\infty} ds W(s) \frac{1}{\pi} [\text{Im}\Pi^{\text{PH}}(s) - \text{Im}\Pi^{\text{OPE}}(s)] = 0, \quad (5)$$

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where $W(s)$ is an arbitrary weight function being analytic except for the positive real axis starting from the lower mass squared, m_c^2 . According to the most successful application of sum rules to mesons and baryons[16], the phenomenological spectral function is assumed to be saturated by one narrow-width resonance and a continuum

$$\frac{1}{\pi} \text{Im}\Pi^{\text{PH}}(s) = F\delta(s - m_R^2) + \frac{1}{\pi} \text{Im}\Pi^{\text{OPE}}(s)\theta(s - s_0), \quad (6)$$

where s_0 is the QCD continuum threshold, the pole residue is of the form $F = f_R^2 m_R^{2k}$ with f_R being the couplings of the lowest resonances with respective parities to the hadronic currents and m_R a pole mass. The power k of m_R^2 in the pole residue is taken to match the maximum power of s in the asymptotic s -behavior of the spectral function. For $s > s_0$, the hadronic continuum reduces to the same form with that obtained by an analytic continuation of the OPE[11], i.e. the perturbative terms, based on a hypothesis of the quark-hadron duality.

Implementing the Borel transformation to the QSRs and performing the OPE at dimension $d \leq 6$ operators, the relations of the BSRs of the lowest 0^\pm $c\bar{s}$ -mesons are obtained to be [11]

$$\begin{aligned} & f_{0^\pm}^2 m_{0^\pm}^2 e^{-m_{0^\pm}^2 \sigma} \\ &= \frac{3}{8\pi^2} \int_{m_c^2}^{s_{0^\pm}} ds e^{-s\sigma} s \left(1 - \frac{m_c^2}{s}\right)^2 \\ &\times \left(1 \mp \frac{2m_c m_s}{s - m_c^2} + \frac{4}{3} \frac{\alpha_s(s)}{\pi} R_0(m_c^2/s)\right) \\ &+ e^{-m_c^2 \sigma} \left[\pm m_c \langle \bar{s}s \rangle_0 + \frac{1}{2}(1 + m_c^2 \sigma) m_s \langle \bar{s}s \rangle_0 \right. \\ &+ \frac{1}{12} \left(\frac{3}{2} - m_c^2 \sigma\right) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0 \\ &+ \left(\pm \frac{\sigma}{2} \left(1 - \frac{m_c^2 \sigma}{2}\right) m_c - \frac{m_c^4 \sigma^3}{12} m_s\right) \langle \bar{s}g\sigma \cdot Gs \rangle_0 \\ &- \frac{16\pi\sigma}{27} \left(1 + \frac{m_c^2 \sigma}{2} - \frac{m_c^4 \sigma^2}{12}\right) \alpha_s \langle \bar{s}s \rangle_0^2 \Big] \\ &- e^{s_{0^\pm} \sigma} \left[\pm m_c \langle \bar{s}s \rangle_0 + \frac{m_s \langle \bar{s}s \rangle_0}{2} + \frac{1}{8} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0 \right] \\ &= M(\sigma), \end{aligned} \quad (7)$$

for the scalar 0^+ and pseudoscalar 0^- channels respectively, where g denotes strong coupling constant, $G^2 = G_{\mu\nu} G^{\mu\nu}$ with $G_{\mu\nu}$ being the gluon field and $\sigma \cdot G = \sigma_{\mu\nu} G^{\mu\nu}$, and $\langle O \rangle_0$ represents the vacuum expectation value of a local composite operator $O(0)$ at the origin. Here, m_s -correction is maintained at the first order [11], and the α_s correction to the perturbative contribution is given as the functions, $R_0(x)$, as follows[18, 19]

$$\begin{aligned} R_0(x) &= \frac{9}{4} + 2Li_2(x) + \ln x \ln(1-x) - \frac{3}{2} \ln \frac{1-x}{x} \\ &- \ln(1-x) + x \ln \frac{1-x}{x} - \frac{1-x}{x} \ln x, \end{aligned} \quad (8)$$

with the Spence function $Li_2(x) = -\int_0^x dt t^{-1} \ln(1-t)$. The running coupling constant $\alpha_s(s)$ appearing in the perturbative terms of Eq.(7) is approximated by a one-loop form [11], $\alpha_s(s) = 4\pi/[9 \ln(s/\Lambda_{QCD}^2)]$ with $\Lambda_{QCD}^2 = (0.25 \text{ GeV})^2$, which is determined to reproduce $\alpha_s(1 \text{ GeV}) \simeq 0.5$ [20]. We note here that the contributions above the threshold s_0 on both sides of Eq.(8) are assumed to be equal to each other due to the quark-hadron duality at high scale, and have been removed out. In fact, our expression of Eq.(8) is different from Ref.[11], namely, some condensate contributions which remain finite above s_0 is thrown off besides the perturbative contribution (s. the last line in Eq.(8)).

In order to derive the GSRs, we make the Laplacian transform on Eq.(7) with the formula[21]

$$\begin{aligned} & \frac{1}{2\tau} \hat{L} \left[\frac{1}{\sigma} e^{-(s+\hat{s})\sigma} M(\sigma) \right] \\ &= \frac{1}{\sqrt{4\pi\tau}} \int_0^\infty ds e^{-\frac{(s+\hat{s})^2}{4\tau}} \frac{1}{\pi} \text{Im}\Pi(s) = G(-\hat{s}, \tau). \end{aligned} \quad (9)$$

This gives a procedure to construct $G(-\hat{s}, \tau)$ from Eq(7), and hence $G(\hat{s}, \tau)$ by analytic continuation. The resultant expressions of GRSs for the $D_s(0^\pm)$ meson currents are

$$\begin{aligned} & f_{0^\pm}^2 m_{0^\pm}^2 \exp \left[-\frac{(m_{0^\pm}^2 - \hat{s})^2}{4\tau} \right] \\ &= \frac{3}{8\pi^2} \int_{m_c^2}^{s_{0^\pm}} ds \exp \left[-\frac{(s - \hat{s})^2}{4\tau} \right] s \left(1 - \frac{m_c^2}{s}\right)^2 \\ &\times \left[1 \mp \frac{2m_c m_s}{s - m_c^2} + \frac{4}{3} \frac{\alpha_s(s)}{\pi} R_0(m_c^2/s)\right] \\ &+ \exp \left[-\frac{(m_c^2 - \hat{s})^2}{4\tau} \right] \cdot \left[\pm m_c \langle \bar{s}s \rangle_0 \right. \\ &+ \frac{1}{2} \left(1 + \frac{2(m_c^2 - \hat{s})m_c^2}{4\tau}\right) m_s \langle \bar{s}s \rangle_0 \\ &+ \frac{1}{12} \left(\frac{3}{2} - \frac{2(m_c^2 - \hat{s})m_c^2}{4\tau}\right) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0 \\ &\pm \frac{1}{2} \left(\frac{3m_c^2 - 2\hat{s}}{4\tau} - \frac{2(m_c^2 - \hat{s})^2 m_c^2}{(4\tau)^2}\right) m_c \langle \bar{s}g\sigma \cdot Gs \rangle_0 \\ &+ \left(\frac{(m_c^2 - \hat{s})m_c^4}{(4\tau)^2} - \frac{2(m_c^2 - \hat{s})^3 m_c^4}{3(4\tau)^3}\right) m_s \langle \bar{s}g\sigma \cdot Gs \rangle_0 \\ &- \frac{16\pi}{27} \left(\frac{m_c^2 - 2\hat{s}}{4\tau} + \frac{(m_c^2 - \hat{s})m_c^2(3m_c^2 - 2\hat{s})}{(4\tau)^2} \right. \\ &- \left. \frac{2(m_c^2 - \hat{s})^3 m_c^4}{3(4\tau)^3}\right) \alpha_s \langle \bar{s}s \rangle_0^2 \Big] - \exp \left[-\frac{(s_{0^\pm} - \hat{s})^2}{4\tau} \right] \\ &\times \left[\pm m_c \langle \bar{s}s \rangle_0 + \frac{m_s \langle \bar{s}s \rangle_0}{2} + \frac{1}{8} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0 \right] \\ &= G(\hat{s}, \tau), \end{aligned} \quad (10)$$

in the scalar 0^+ and pseudoscalar 0^- channels, respectively.

Taking the derivative with respect to \hat{s} for both sides

of Eq.(10), we get

$$f_{0\pm}^2 m_{0\pm}^2 (m_{0\pm}^2 - \hat{s}) e^{-\frac{(m_{0\pm}^2 - \hat{s})^2}{4\tau}} = 2\tau \frac{\partial G(\hat{s}, \tau)}{\partial \hat{s}}. \quad (11)$$

From Eqs.(10) and (11), we obtain the sum rules for the masses of lowest resonances and couplings to the corresponding currents

$$m_{0\pm}^2 = \hat{s} + \frac{2\tau}{G(\hat{s}, \tau)} \frac{\partial G(\hat{s}, \tau)}{\partial \hat{s}}, \quad (12)$$

$$f_{0\pm}^2(s; \hat{s}, \tau) = \frac{G(\hat{s}, \tau)}{m_{0\pm}^2} \exp \left[\frac{(m_{0\pm}^2 - \hat{s})^2}{4\tau} \right]. \quad (13)$$

In order to extract the values of physical quantities from the GSRs, we use the following standard values for QCD parameters appearing in the OPE[11](see Tab.I).

TABLE I: QCD input parameters used in the analysis.

Parameters	References
$m_s = 0.11 \pm 0.01$ GeV	[16]
$m_c = 1.46$ GeV	[13]
$\langle \bar{n}n \rangle_0 = (-0.225 \pm 0.025 \text{ GeV})^3$	[16]
$\langle \bar{s}s \rangle_0 = (0.8 \pm 0.1) \times \langle \bar{n}n \rangle_0 \text{ GeV}^3$	[16]
$\langle \frac{\alpha_s}{\pi} G^2 \rangle_0 = (0.33 \text{ GeV})^2$	[18]
$\langle \bar{n}g\sigma \cdot Gn \rangle_0 = M_0^2 \langle \bar{n}n \rangle_0$	[22]
$M_0^2 = 2 \times (0.4 \pm 0.1) \text{ GeV}^2$	[22]
$\langle \bar{s}g\sigma \cdot Gs \rangle_0 = M_0^2 \langle \bar{s}s \rangle_0$	
$\alpha_s \langle \bar{n}n \rangle_0^2 = 0.162 \times 10^{-3} \text{ GeV}^6$	
$\alpha_s \langle \bar{s}s \rangle_0^2 = (0.8 \pm 0.1)^2 \times \alpha_s \langle \bar{n}n \rangle_0^2 \text{ GeV}^6$	

For numerical calculation, we must determine the values of \hat{s} and the thresholds s_0 . To investigate the properties of the considered resonance, the value of \hat{s} should approximately be set to be the corresponding mass squared, $m_{D_s}^2$, of the resonance. To suppress the continuum contribution, we require $\hat{s} \leq m_{D_s}^2$. The conditions for determine the value of s_0 are: first, it should be grater than $m_{D_s}^2$; second, it should guarantee that there exists a sum rule window for our sum rules. We note that the upper limit of the sum rule window is determined by requiring the contribution of continuum to be lower than 30% of the total, while the lower limit of that window is obtained by requiring the non-perturbative contributions, proportional to a positive powers of σ , to be less than 30% (in fact, less than 10%) of the perturbative one. Therefore, the value of s_0 , the upper and lower limits of the corresponding sum rule window are determined in self-consistent manipulation.

Then, using the GSRs of the mass and coupling constant of $D_s(0^\pm)$, we can get the figures and numerical results shown below. Figs. 1 and 2 display the dependencies of the calculated masses $m_{D_s(0^\pm)}$ on τ from GSRs, Figs. 3 and 4 plot the coupling $f_{D_s(0^\pm)}$ vs τ . In Figs. 1 and 2, the thick horizontal-lines on the curves are the

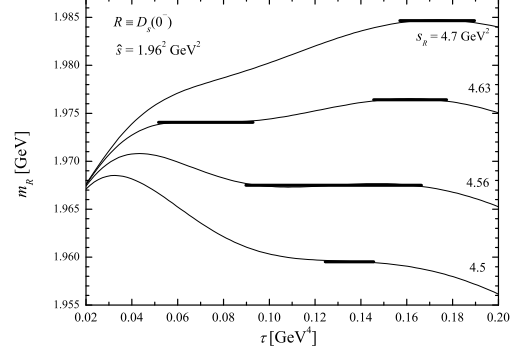


FIG. 1: The curves of $m_R \equiv m_{D_s(0^-)}$ vs. τ from GSRs, where $\hat{s} = 1.96^2 \text{ GeV}^2$, $s_{D_s(0^-)} = 4.5 \sim 4.7 \text{ GeV}^2$.

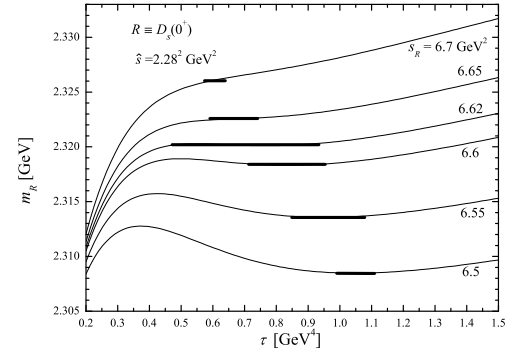


FIG. 2: The Gauss curves of $m_R \equiv m_{D_s(0^+)}$ vs. τ , where $\hat{s} = 2.28^2 \text{ GeV}^2$, $s_{D_s(0^+)} = 6.5 \sim 6.7 \text{ GeV}^2$.

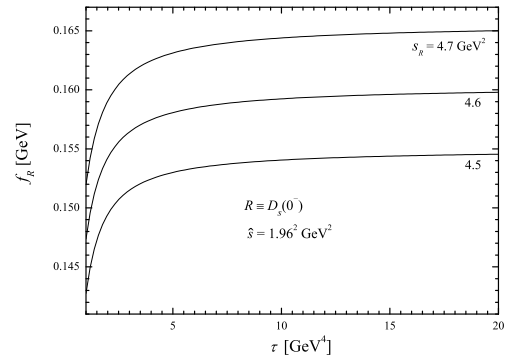


FIG. 3: The Gauss curves of $f_R \equiv f_{D_s(0^-)}$ vs. τ , where $\hat{s} = 1.96^2 \text{ GeV}^2$, $s_{D_s(0^-)} = 4.5 \sim 4.7 \text{ GeV}^2$.

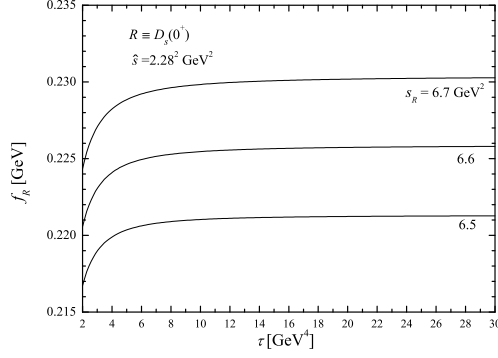


FIG. 4: The Gauss curves of $f_R \equiv f_{D_s(0^+)}$ vs. τ , where $\hat{s} = 2.28^2 \text{ GeV}^2$, $s_{D_s(0^+)} = 6.5 \sim 6.7 \text{ GeV}^2$.

Gauss stabilities of the sum rule windows. The length of the lines corresponds to the sizes of stable regions.

In $D_s(0^-)$ channel with $\hat{s} = 1.96^2 \text{ GeV}^2$, we cannot find any Gaussian stability below $s_{0^-} = 4.5 \text{ GeV}^2$ and above $s_{0^-} = 4.7 \text{ GeV}^2$, while between these two values the curves are stable. From these plateau regions, we get the resonance mass, $m_{D_s(0^-)}$, to be $1.968 \pm 0.016 \pm 0.003 \text{ GeV}$, in which, ± 0.016 is the error from theory (here, we mean that the upper and lower values of $m_{D_s(0^-)}$ are determined by the upper and lower limits of s_{0^-} , as done in Ref.[11]) and ± 0.003 is the error from input parameters. We can see that the result of GSR is larger than the value of BSR[11], $1.94 \pm 0.03 \text{ GeV}$, and closer to the experimental value 1.969 GeV (s. Table II). The values of $f_{D_s(0^-)}$, $m_{D_s(0^+)}$ and $f_{D_s(0^+)}$ are obtained in a similar way. The Numerical results are listed in Table II. We can see from Table II clearly that the results of GSRs are in accordance well with experiment than others.

For checking the self-consistency of GSRs, we also compare the l.h.s. of Eq(10) with the r.h.s., using the center values determined from GSRs. We have found that the two sides of GSRs, Eq. (10), are compatible from each other very well.

TABLE II: The numerical results of QCD sum rules. For comparison, we attach experimental average values observed [4–9] and the masses of the first radial excitations predicted in Ref.[12]. (GeV)

m_R	(GSR)	(BSR)[11]	(exp.)	(model)[12]
0^-	$1.968 \pm 0.016 \pm 0.003$	1.94 ± 0.03	1.969	2.700
0^+	$2.320 \pm 0.014 \pm 0.003$	2.48 ± 0.03	2.317	3.067
f_R	f_R (GSR)			
0^-	$0.158 \pm 0.006 \pm 0.003$			
0^+	$0.225 \pm 0.005 \pm 0.003$			

As a summary, in the framework of the Gaussian sum rules, the masses of the $D_s(0^\pm)$ mesons are investigated from a view-point of ordinary light-heavy system. The GSRs for the masses of $D_s(0^\pm)$ mesons and the couplings of $D_s(0^\pm)$ mesons to the corresponding currents are derived by means of the Laplacian transformation to the usual Borel sum rules. Using the standard input of QCD non-perturbative parameters, the corresponding mass spectra and couplings of the currents to the $D_s(0^\pm)$ mesons are obtained. By comparing both sides of the GSRs, we have shown that there exists a stability regions within which both sides of the sum rules are matched very well. Our results are $m_{D_s(0^-)} = 1.968 \pm 0.016 \pm 0.003 \text{ GeV}$ and $m_{D_s(0^+)} = 2.320 \pm 0.014 \pm 0.003 \text{ GeV}$, which are in accordance well with the experimental data, 1.969 GeV and 2.317 GeV . Finally, it worth noting that the $D_s^{*\pm}(2317)$ is treated as the lowest resonance of the 0^+ $c\bar{s}$ meson in our calculation, and furthermore, its mass is even lower than that of the 0^+ $c\bar{d}$ meson estimated in the similar way (this result will be published elsewhere). Therefore, $D_s^{*\pm}(2317)$ may be considered to be the lowest 0^+ charmed meson in our treatment.

Acknowledgements

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